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Qualitative Economics: An Implementation in PROLOG

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Abstract. In this paper, we describe a formalism for qualitative reasoning in economics. The framework may serve as a common basis for the intuitive reasoning practised by experienced economists and the more formal qualitative models recently established in the field of artificial intelligence. The emphasis is on representation and implementation aspects of qualitative models. The formalism is illustrated in a well-known Keynesian model.

Key words. Qualitative reasoning, expert systems, macroeconomics.

1. Introduction

The developments in qualitative modelling originate mainly from AI research in the field of 'naive' physics (Forbus, 1986; Hayes, 1979; de Kleer and Brown, 1984; Kuipers, 1986). The aim of 'naive' physics is to provide a description of physical devices in qualitative terms in such a way that only crucial distinctions in qualitative behaviour are preserved. In addition, there are also some contributions in the fields of electronic circuit analysis (Davis, 1984; Genesereth, 1984) and medical diagnosis (Chandrasekaran and Mittal, 1983; Kuipers and Kassirer, 1984). Recently, researchers have started to consider the exploration of these ideas in economics which has led to some interesting results. For example, the differences between quantitative and qualitative models are illustrated by the classical macro-economic theory of output and employment in Farley (1986). Constraint propagation techniques are applied to a model concerning the equilibrium of the commodity and labour market in Bourgine and Raiman (1986). In Pau (1986), qualitative arguments occurring in government texts addressing economic subjects are mapped into a formal grammar.

Application of AI methods undoubtedly contributed to the understanding of economic reasoning. However, some of the underlying ideas in qualitative modelling have already been published in the economic literature. The similarity between the theory of confluences (de Kleer and Brown, 1984) and comparative statics (Samuelson, 1947) is pointed out in Iwasaki and Simon (1986a) and a profound treatment of qualitative statics can be found in Greenberg and Maybee (1981).

The reexploration in the application of formal qualitative modelling in economics is mainly due to the tremendous increase in computer power and the proliferation of symbolic programming languages such as LISP and PROLOG. One of the intrinsic reasons for studying qualitative methods is the lack of consistent data that are indispensable in quantitative models. A practical reason is the intractability of the huge amounts of computer output of complex numerical models. Other reasons are the wish to create automatic procedures for tracing causal chains, and to provide procedures to support the validation of the structure of economic models (Boutillier, 1984; Fontela, 1986; Gallo and Gilli, 1990; Royer and Ritschard, 1984). In any case, we believe that qualitative modelling provides a way of filling the gap between number crunching programs and verbal intuitive reasoning (Paliès and Philip, 1989; Berndsen and Daniels, 1989a).

In this paper, we propose a constraint oriented approach to qualitative modelling. The method can be positioned somewhere between the theory of qualitative reasoning based on confluences (de Kleer and Brown, 1984) and qualitative simulation (QSIM) (Kuipers, 1986). The main differences emerge from the fact that these theories were developed to study the qualitative behaviour of *physical* systems. In the formalism proposed in this paper, qualitative dynamic models consist of standard symbolic constraints (e.g. originating from balance sheet equations), constraints representing contemporaneous causality (if two economic entities influence each other directly) and sequential causality (if the influence is unidirectional and there is a time lag involved (Hicks, 1979)).

Explicit modelling of causality seems quite natural in economics. In earlier papers (Iwasaki and Simon, 1986a, 1986b; de Kleer and Brown, 1986), causal relations are derived from a static mathematical model. It can be shown that causality derived from static models by the methods of causal ordering or mythical causality does not reflect the intuitive notion of causality (Iwasaki, 1988; Berndsen and Daniels, 1989a). One way to get around this problem is to consider dynamic models (Iwasaki, 1988). However, we believe that dynamic models possess a level of detail which is unnecessary to describe the qualitative behaviour of economic systems. Therefore, we start from a declarative representation of causality based on economic theory. Similar ideas have been considered in the description of physical devices (Rieger and Grinberg, 1977).

In Section 2, the formal semantics of the constraint language is described. An implementation in PROLOG is discussed in Section 3. In Section 4, a Keynesian model is presented.

2. Qualitative Modelling

2.1. FORMALISM

In the following, an *economic system* $\mathcal{S}$ is defined as:

(i) a set of variables $V = \{v_j\} (j = 1, \ldots, n)$,

(ii) a set of quantity spaces QSval_j and QSdir_j for every variable v_j,
(iii) a set of constraints C.

Furthermore, time is represented by a finite set of half-open time intervals of uniform length:

$$T = \{[t_0, t_1\rangle, \ldots, [t_{n-1}, t_n\rangle\} = \{i_0, \ldots, i_{n-1}\} .$$

For every variable v_j, two functions on T are defined:

$\mathrm{Qval}(v_j)$: $T \rightarrow \mathrm{QSval}_j$ denoting the *qualitative value* of v_j at $t_k \in T$ and
$\mathrm{Qdir}(v_j)$: $T \rightarrow \mathrm{QSdir}_j$ denoting the *qualitative direction of* v_j in $i_k \in T$.

The interpretation is that $\mathrm{Qval}(v_j)$ is defined at the beginning of time interval i_k, i.e. the time point t_k and $\mathrm{Qdir}(v_j)$ is defined in the complete interval. QSval_j and QSdir_j are called quantity spaces.

A *quantity space* is a totally ordered finite set of symbolic values. Various quantity spaces have been proposed in the literature (de Kleer and Brown, 1984; Kuipers, 1986; Raiman, 1986). Here we take for Qdir the quantity space $\mathrm{QSdir}_j = \{\mathrm{inc}, \mathrm{std}, \mathrm{dec}\}$ and for Qval either $\mathrm{QSval}_j = \{+, 0, -\}$ or $\mathrm{QSval}_j = \{\lambda\}$. In the first case, the set $\{+, 0, -\}$ denotes the relative position of a variable v_j with respect to an important (landmark) value l. For example, if v_j denotes excess demand, l could be the value of v_j at which the corresponding market is in equilibrium. In the second case, QSval_j is restricted to a single element $\{\lambda\}$ which may denote $\langle -\infty, \infty\rangle$ or $\langle 0, \infty\rangle$. This is the quantity space for variables for which only the qualitative direction is of importance.

A *qualitative* state $\mathrm{QS}(v_j, i_k)$ of a variable v_j at i_k is defined as the pair $(\mathrm{Qval}(v_j, t_k), \mathrm{Qdir}(v_j, i_k))$. A qualitative state of an economic system $\mathcal{S}$ at i_k is the list of qualitative states of the n variables v_j:

$$\mathrm{QS}(\mathcal{S}, i_k) = \mathrm{QS}(v_1, i_k), \ldots, \mathrm{QS}(v_n, i_k) .$$

An *admissible* qualitative state $\mathrm{QS}(\mathcal{S}, i_k)$ is a qualitative state of $\mathcal{S}$ such that all constraints are satisfied simultaneously. The corresponding assignment of qualitative states to all variables is called a *valid interpretation*.

Constraints are relations among variables that impose restrictions on combinations of Qval's or Qdir's of the variables in the constraint. There are several types of constraints. Some constraints correspond to familiar mathematical operators, such as addition and differentiation, in a qualitative context. Other constraints define monotonic and causal relationships between variables. The constraints are defined in Subsection 2.2.

A *valid state transition* is an ordered pair $\mathrm{QS}(\mathcal{S}, i_k)$, $\mathrm{QS}(\mathcal{S}, i_{k+1})$ of admissible states such that $\forall j$ $\mathrm{QS}(v_j, i_k)$, $\mathrm{QS}(v_j, i_{k+1})$ is a valid variable transition. The set of valid variable transitions can be divided into two disjoint subsets QD and QS (listed in Tables I and II, respectively). The set of QD-transitions is relevant if $\mathrm{QSval}_j = \{\lambda\}$, i.e. only transitions of the qualitative direction are taken into account. Otherwise, $\mathrm{QSval}_j = \{+, 0, -\}$ and so-called QS-transitions apply.

Table I. QD-transitions[a]

	$\mathrm{Qdir}(x_i, i_k)$	$\rightarrow$	$\mathrm{Qdir}(x_i, i_{k+1})$
QD1	Any		std
QD2	Any		inc
QD3	Any		dec

[a]where each QDi is a subset of 3 transitions with Any $\in$ QSdir = {inc,std,dec}. Analogously, QSi is a subset of 3 or 9 (QS5 and QS8) transitions.

Table II. QS-transitions[a]

	$\mathrm{QS}(x_i, i_k)$	$\rightarrow$	$\mathrm{QS}(x_i, i_{k+1})$
QS1	(0,std)		(0,Any)
QS2	(0,inc)		(+,Any)
QS3	(0,dec)		(−,Any)
QS4	(+,dec)		(0,Any)
QS5	(+,Any)		(+,Any)
QS6	(+,dec)		(−,Any)
QS7	(−,inc)		(0,Any)
QS8	(−,Any)		(−,Any)
QS9	(−,inc)		(+,Any)

A *qualitative behaviour* of a variable v_j from i_k to i_{k+n} is a sequence of qualitative states with valid transitions between them:

$$\mathrm{QS}(v_j, i_k), \ldots, \mathrm{QS}(v_j, i_{k+n}) .$$

Accordingly, a qualitative behaviour of the system $\mathcal{S}$ from i_k to i_{k+n} is the corresponding sequence of admissible qualitative states of $\mathcal{S}$.

The *envisionment* of $\mathcal{S}$ with initial state $\mathrm{QS}(\mathcal{S}, i_0)$ is a rooted directed graph E with the following properties:

(a) $\mathrm{QS}(\mathcal{S}, i_0)$ is the root,
(b) the set of nodes of E contains all admissible qualitative states of $\mathcal{S}$ that are reachable from the root by valid state transitions,
(c) there is a link between two nodes of E iff there exists a valid state transition between them.

A path from the root to another node corresponds to some qualitative behaviour of the system.

2.2. CONSTRAINTS

In the following, we describe the restrictions induced on the economic variables by each constraint. In this paper, constraints express qualitative relations between economic variables. These are analogous to equations in a quantitative model. The set of constraints is specified as part of the modelling process on the basis of economic theory or heuristic economic knowledge. There are several types of constraints. Here, the discussion is limited to the types of constraints of the Keynesian example presented in Section 4. The ADD-constraint specifies the qualitative analogue of an accounting identity, whereas M^+, DERIV and SC^+ represent behavioural laws of different nature.

2.2.1. *ADD-constraint*

ADD(a, b, c) defines the variable c as the qualitative sum of the variables a and

b. Depending on the particular application at hand, it is possible to take both Qval and Qdir into account or only Qdir.

The former case applies only if for all variables joined by an ADD-constraint QSval $= \{+, 0, -\}$. It is assumed that the ADD-constraint holds for the tuple $(0, 0, 0)$. A tuple of qualitative values of the variables a, b and c satisfy the ADD-constraint at i_k if:

$$\text{Qval}(a, i_k) \oplus \text{Qval}(b, i_k) \cong \text{Qval}(c, i_k)\,,$$

where $\oplus$ (qualitative addition) and the weak equality sign $\cong$ are defined by the following tables:

$\oplus$	+	−	0	?
+	+	?	+	?
−	?	−	−	?
0	+	−	0	?
?	?	?	?	?

$\cong$	+	−	0	?
+	T	F	F	T
−	F	T	F	T
0	F	F	T	T
?	T	T	T	T

The weak equality sign $\cong$ is a two-place predicate. Here we will not go into details of qualitative algebra, the interested reader is referred to Dormoy and Raiman (1988), Travé-Massuyès and Piera (1989), and Williams (1988).

Furthermore, the ADD-constraint puts a restriction on the Qdir's of a, b and c, which is equivalent to the restriction on the Qval's.

2.2.2. *M^+ and M^--constraint*

The monotonicity constraints $\text{M}^+(a, b)$ and $\text{M}^-(a, b)$ define a monotonic functional relationship between a and b. M^+ is appropriate if the relationship between a and b is monotonic and increasing. Conversely, if the relationship is decreasing and monotonic, the M^--constraint applies. The monotonicity constraint puts a restriction on the Qdir's of a and b, namely for the M^+-constraint: $\text{Qdir}(a, i_k) = \text{Qdir}(b, i_k)$, and similarly with a minus sign for M^-. This corresponds to the formal representation of contemporaneous causality defined in Hicks (1979).

2.2.3. *DERIV-constraint*

The qualitative analogue of the derivative relation between two variables a and b is represented by DERIV(a, b) where b is the qualitative time derivative of a. Intuitively, if b is positive at t_k, then a is increasing at i_k (analogously if $b \leqslant 0$). DERIV(a, b) is satisfied at i_k iff the pair (Qdir(a, i_k), Qval(b, t_k)) matches one of the entries in the table below. The quantity space QSval(b) must equal $\{+, 0, -\}$.

DERIV(a, b)	Qdir(a, i_k)	Qval(b, t_k)
	std	0
	inc	+
	dec	−

2.2.4. *SC^+ – and SC^--constraint*

The causal constraints $SC^+(a, b)$ and $SC^-(a, b)$ denote the relation of sequential causality between a and b (Hicks, 1979). $SC^+(a, b)$ holds if a influences b positively. If the influence of a on b is negative, then $SC^-(a, b)$ holds. The constraint $SC^+(a, b)$ puts a restriction on the pair $(QS(a, i_{k-1}), QS(b, i_k))$ as follows:

$$\mathrm{Qdir}(a, i_{k-1}) = \mathrm{Qdir}(b, i_k) \text{ and similarly with a minus sign for } SC^-.$$

3. Implementation

In this section, we outline an implementation of the envisionment algorithm in PROLOG. In the text, the code is described in a maximal declarative form which makes it simple and readable but inefficient. The actual source code* differs considerably from the code described below because of efficiency reasons.

3.1. INPUT

The input of the envisionment algorithm consists of an economic system $\mathcal{S}$ and an initial state $QS(\mathcal{S}, i_0)$ (see Section 2.1). An economic system is given by the predicate: `economic_system(Name, var(V), qsval(Q), constraints(C))`. Name denotes the name of the system and `var(V)` represents the economic variables V. The third argument denotes the quantity-space QSval_j of variable j. The last argument species the set of constraints C.

A qualitative state $QS(\mathcal{S}, i_k)$ is represented by the clause `qs(N, i(K), Qs)`. The first argument N identifies the state uniquely. The second argument denotes the time-interval i_k and Qs is the list of qualitative states of V. The elements of Qs are $[v, \mathrm{Qval}, \mathrm{Qdir}]$, where v is an economic variable. The initial state is represented as `qs(n1, i(0), Qs_initial)`.

3.2. ENVISIONMENT

The envisionment of the economic system is generated breadth-first by the following predicate:

* An implementation in LPA-PROLOG is available at the authors' address.

```
envisionment([N1|Open]):-
  admissible_successors(N1,Suc),
  concat_new(Open,Suc,Newsuc),
  envisionment(Newsuc).
envisionment([]).
```

N1 is the node of the graph to expand. Open is a list of nodes to be expanded later on. Suc is the list of admissible successors of N1. The envisionment of the list [N1|Open] is recursively defined as the list of admissible successors of N1 and the envisionment of the list Newsuc of open nodes. The predicate concat_new ensures that only new qualitative states (relative to the current set of nodes) are concatenated to Open.

The predicate admissible_successors determines all admissible successors N2 of node N1:

```
admissible_successors(N1,Suc):-
  findall(N2,admissible_successor(N1,N2),Suc).
```

A node N2 is an admissible successor of N1 if there exists a valid state transition from N1 to N2 and N2 is an admissible state:

```
admissible_successor(N1,N2):-
  valid_state_transition(N1,N2),
  admissible_state(N2).
```

A state transition is valid if there is a valid variable transition for all variables of the economic system (Section 2.1). The validity of variable transitions is checked by table lookup (see Tables I and II).

A qualitative state is admissible if all constraints of the economic system are satisfied:

```
admissible_state(N2):-
  economic_system(_,_,_,constraints(C)),
  valid_interpretation(C,N2).
```

The predicate valid_interpretation checks if all constraints are satisfied, i.e. N2 is a valid interpretation. The technique of determining a valid interpretation is a constraint satisfaction problem.

3.3. CONSTRAINT SATISFACTION

A state N2 is a valid interpretation if all tuples of qualitative values are valid tuples i.e. satisfy the appropriate constraints. This is checked by filtering every constraint. The two-place predicate valid_interpretation is given by:

```
valid_interpretation([C|Tail],N2):-
  filter(C,Valid_tuple),
  compatible(Valid_tuple,N2),
  valid_interpretation(Tail,N2).
valid_interpretation([],_).
```

`[C|Tail]` is the list of constraints to solve. The predicate `filter(C,Valid_tuple)` filters constraint C i.e. it determines whether a tuple of qualitative states of variables called `Valid_tuple` satisfies C. For every type of constraint a separate clause is defined that specifies the tuples of qualitative states for which the constraint is satisfied. These clauses correspond to the constraint semantics discussed in Section 2.2. The predicate `compatible(Valid_tuple,N2)` checks if N2 agrees with `Valid_tuple`.

Remark. In the description of the clause `valid_interpretation`, above, constraint satisfaction is stated in its pure declarative form, omitting all computational details. Constraint satisfaction problems occur in many different contexts and there is a considerable interest in efficient algorithms for solving them, see, e.g., Van Hentenryck (1989). In Kuipers (1986), the constraint satisfaction process is done in three steps: constraint consistency filtering, pairwise consistency filtering, and global consistency filtering. In the first step, assignments of qualitative states to variables in a particular constraint are made in such a way that the constraint is satisfied. For every constraint C_i the assignments are gathered in a list L_i. In the second step, pairs of adjacent constraints (C_i, C_j) $(i \neq j)$ are considered. A pair of constraints is adjacent if they share one or more variables. Tuples on L_i in which a qualitative state to a common variable is assigned that is not in any tuple of L_j, are deleted. This process terminates if no more tuples can be deleted from any list L_i. In the third step, all constraints are considered together. A valid interpretation is obtained by selecting a tuple from every list L_i such that all occurrences of the same variable are assigned the same qualitative state.

To understand the constraint satisfaction process in the PROLOG code described here, the clauses should be read in a procedural way. The constraint satisfaction technique employed in the actual implementation uses backtracking over constraints. Initially, none of the variables of N2 is instantiated with a qualitative state. Backtracking occurs whenever a tuple of variables of a constraint is not consistent with N2.

3.4. BEHAVIOUR

The envisionment of an economic model is the description of all possible behaviours of the model. Among these behaviours only a few categories are interesting. Economists usually look for tendencies towards equilibrium or unstable paths. In the following we define three types of 'interesting' behaviour. To do this, two notions of stationary states are defined: equilibrium and no_change states. An equilibrium state is a state with Qdir(v_i) = std for every variable v_i. A no_change state is a state that is a successor of itself.

Equilibrium behaviour is a path from the root to an equilibrium state. No_change behaviour is a path from the root to a no_change state. Finally, cyclic behaviour is a cyclic path. We restrict cyclic paths to distinct elementary cycles only, because all cycles can be thought of as composed of elementary cycles (an elementary cycle is a path where no node but the first and last appears twice and two cycles are distinct if one is not a cyclic permutation of the other).

The first two types of behaviour are found easily. The third type of behaviour is found by employing a well-known algorithm that enumerates all distinct elementary cycles of a directed graph (cf. Mateti and Deo, 1976). An efficient version of this algorithm is developed and implemented in Algol W by Johnson (1975). The algorithm in PROLOG reads ony a few lines of code.

4. Example

In this section, we present some results of the application of the envisionment algorithm to a simple Keynesian model.

4.1. THE MODEL

The Keynesian model consists of the following constraints:

(1) ADD(c,i,y)	(2) ADD($m1,m2,md$)	(3) ADD(emd,ms,md)
(4) DERIV(r,emd)	(5) $M^+(m1,y)$	(6) $M^-(r,m2)$
(7) $SC^+(y,c)$	(8) $SC^-(r,i)$.	

where c = consumption, $m1$ = transactions money demand,
i = investment, $m2$ = speculative money demand,
y = national income, md = total money demand,
r = interest, ms = money supply,
emd = excess money demand.

Constraint (1) denotes the national accounting identity in a closed economy without a government and (2) defines total money demand as the sum of $m1$ and $m2$. The third constraint defines excess money demand as the differences between money demand and money supply. The money supply is an exogenous variable controllable by the monetary authorities. Constraint (4) represents the adjustment mechanism of the money market. The money demand relationships are modelled by monotonicity constraints (5, 6). Constraints (7) and (8) state that consumption depends on income and investment on interest.

The quantity space QSval of emd is $\{+, 0, -\}$ where '0' corresponds to money market equilibrium. The other variables have QSval = $\{\lambda\}$.

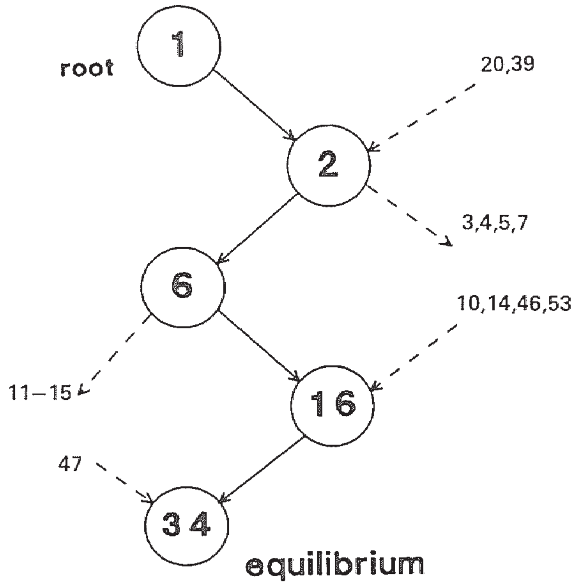

Fig. 1. A part of the envisionment graph of the Keynesian model.

4.2. THE ENVISIONMENT GRAPH

The root of the envisionment graph represents a situation where a positive money supply shock is given in time interval i_0.

The complete envisionment of the Keynesian model consists of 54 states. The part of the envisionment graph corresponding to equilibrium behaviour is depicted in Figure 1. Dotted arrows denote incoming and outgoing links of states not shown. The corresponding qualitative states are given in Table III.

From Table III may be concluded that the levels of income and consumption in the new equilibrium position are higher compared to the equilibrium before the money supply shock (because y and c never decrease).

However, equilibrium behaviour is but one possibility. In the complete envisionment graph, there are 8 no_change nodes (or 9 including the equilibrium

Table III. Qualitative states of equilibrium behaviour

Node x_j	1 QS(x_j, i_0)	2 QS(x_j, i_1)	6 QS(x_j, i_2)	16 QS(x_j, i_3)	34 QS(x_j, i_4)
c	λ,std	λ,std	λ,inc	λ,inc	λ,std
i	λ,std	λ,inc	λ,inc	λ,dec	λ,std
$m1$	λ,std	λ,inc	λ,inc	λ,std	λ,std
$m2$	λ,inc	λ,inc	λ,dec	λ,std	λ,std
md	λ,inc	λ,inc	λ,dec	λ,std	λ,std
ms	λ,inc	λ,std	λ,std	λ,std	λ,std
emd	−,std	−,inc	+,dec	0,std	0,std
r	λ,dec	λ,dec	λ,inc	λ,std	λ,std
y	λ,std	λ,inc	λ,inc	λ,std	λ,std

Table IV. No change nodes

Node	Qdir(y)	Qdir(r)	QS(emd)	Characterization of the money market
7	inc	dec	−,inc	trend to equilibrium
11	inc	inc	+,std	excess money demand
12	inc	inc	+,inc	trend divergence
14	inc	inc	+,dec	trend to equilibrium
30	dec	inc	+,dec	trend to equilibrium
38	dec	dec	−,std	excess money supply
41	dec	dec	−,inc	trend to equilibrium
42	dec	dec	−,dec	trend divergence

state). These nodes are given in Table IV characterized by the variables y, r and *emd*.

The third type of behaviour is cyclic behaviour. Although the number of cycles is large, it is possible to classify these cycles in a way that is meaningful from an economic point of view. An attempt using the correspondence between the envisionment and the concept of a finite-state automaton can be found in Berndsen and Daniels (1989b).

5. Conclusions

In this paper, a framework is presented for qualitative reasoning in economics. A simple qualitative model serves as an example and different qualitative behaviours are derived in a formal way. Furthermore, some issues about the implementation of this framework in PROLOG are described. Clearly, the analysis here is only a first step and the methods have to be refined considerably to deal with more realistic models.

Future research may enhance the applicability of qualitative reasoning methods in economics. Several lines should be followed. Firstly, one could incorporate other (fixed) quantity spaces and apply order of magnitude reasoning techniques. This will reduce the huge number of nodes in the envisionment graph of complex models, compared to simulations where only sign information is used. Another improvement can be found in the enrichment of the formal language. In this paper economic relations are captured only by simple constraints. The formalism for representing economic relations is poor and should be enriched to contain more sophisticated economic knowledge. Finally, it would be interesting to develop a library consisting of coherent economic structures like markets, that can be combined efficiently to obtain more complex structures.

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